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2-D AND 3-D CELLULAR MODELS USING EROSION AND DILATION: MODEL DEVELOPMENT

Ahmad Faris Ismail
Faculty of Engineering
International Islamic University Malaysia
Lembah Pantai, Jalan Pantai Baru
59200 Kuala Lumpur
Malaysia
Tel.: 603-282-7161
Fax: 603-282-6957
email: faris@eng.iiu.my

ABSTRACT

Based on the erosion and dilation procedures of image processing technique, the three dimensional cellular model is developed and tested. This development is an attempt to solve the main limitation of the corresponding two dimensional model which requires specific assumption on the shape of the particle. In this 3-D method, twenty six-neighbour configuration is investigated and the effect of increasing the threshold values for a particular node is systematically studied.

This method includes the measurement of equivalent radius (for the case of sphere) or length (for the case of cube) and the results are compared with those of two dimensional for different number of threshold values. The 3-D version is shown to be superior since no assumption of particle shape is needed.

Keywords: ***Cellular model, image processing, erosion, dilation***

1.0 INTRODUCTION

Erosion and dilation operations are implemented during digital image processing to perform morphological filtering of the image [1]. The erosion operation is defined as the process to remove (set to white) the black pixels when a chosen number of its neighbours are white while the dilation operation is defined as the process to add (set to black) the white pixels when a chosen number of its eight neighbours are black. The pixel may have up to eight and twenty six neighbours for two dimensional image and three dimensional image, respectively. After the operation, dilation expands an image and erosion shrinks it [2].

In our previous paper [3], we have shown that dissolution processes of a spherical particle can be simulated using two dimensional (2-D) discrete model. Available public domain NIH Image image analysis software (written by Wayne Rasband at the U.S. National Institutes of Health (NIH) and available electronically via Internet by anonymous ftp from zippy.nimh.nih.gov) was used in the simulation.

The basic problem of a 2-D model is that the shape of the particle needs to be assumed [4] and must not change throughout the simulation. However, the shapes normally present in real physical problems are very irregular. The particles do not always take the shape of perfect spheres so that they can be simulated accurately by circles in a 2-D model.

The objective of this paper is to present and discuss both 2-D (using shapes of circle and square) and 3-D (using sphere and cube) cellular models developed in our lab.

2.0 MODEL DEVELOPMENT OF 2-D CELLULAR MODEL

The model begins by initializing the initial shape of the particle. Two shapes, circle and square shown in Fig. 1, are investigated in this paper.

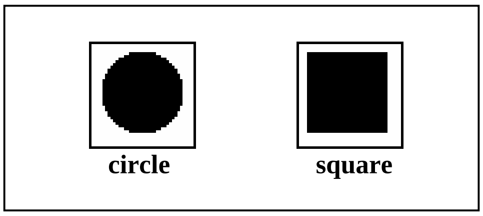

Fig. 1: Initial Shapes For 2-D Model

For each pixel in 2-D model denoted by (i,j), where i and j indicate its position, there are eight neighbours surrounding the pixel: (i-1,j-1), (i-1,j), (i-1,j+1), (i,j-1), (i,j+1), (i+1,j-1), (i+1,j), and (i+1,j+1). Each pixel can only take the value of either 0 (white) or 1 (black).

Erosion iterations are based on the summation (SUM_2D) of the value of neighbouring pixels:

$$\text{SUM\_2D} = \left\{ \sum_{p=i-1}^{i+1} \sum_{q=j-1}^{j+1} value(p,q) \right\} - value(i,j)$$

Threshold values, varied from one to eight, determine whether erosion will occur or not. For example, if value of position (i,j) is 1 (black) and its SUM_2D is less or equal to the threshold then the value at that position (i, j) will be set to 0 (white).

The iterations continue until either there is no more erosion or the image is completely eroded. At each iteration, the area covered by the image as well as its equivalent radius (for the case of circle) or length (for the case of square) are computed and studied.

3.0 MODEL DEVELOPMENT OF 3-D CELLULAR MODEL

The 2-D model described above is extended to three dimension. In 3-D cellular model, each pixel (i,j,k) will have twenty six neighbours:
(i-1,j-1,k-1), (i-1,j-1,k), (i-1,j-1,k+1), (i-1,j,k-1), (i-1,j,k), (i-1,j,k+1),
(i-1,j+1,k-1), (i-1,j+1,k), (i-1,j+1,k+1), (i,j-1,k-1), (i,j-1,k), (i,j-1,k+1),
(i,j,k-1), (i,j,k+1), (i,j+1,k-1), (i,j+1,k), (i,j+1,k+1), (i+1,j-1,k-1), (i+1,j-1,k), (i+1,j-1,k+1), (i+1,j,k-1), (i+1,j,k), (i+1,j,k+1),
(i+1,j+1,k-1), (i+1,j+1,k), and (i+1,j+1,k+1).

The threshold values may vary from 1 to 26 and the sum (SUM_3D) of values of its neighbour is calculated using

$$\text{SUM\_3D} = \left\{ \sum_{p=i-1}^{i+1} \sum_{q=j-1}^{j+1} \sum_{r=k-1}^{k+1} value(pq,r) \right\} - value(i,j,k)$$

Instead of circle and square, we can consider the shapes of sphere and cube for 3-D model. The initial shapes used in this model are shown in Fig. 2 as a function of r which is the distance (in mm) from the centre (for sphere) or the distance from the middle (for cube) on z-coordinate taking other coordinates (x and y) constant. The initial diameter of the sphere and length of cube are set to be 10 mm.

4.0 RESULTS AND DISCUSSION: 2-D MODEL

The results for two dimensional simulation of a circle are shown in Fig. 3 and Fig. 4. In Fig. 3, equivalent diameter, D, defined by

$$D = \sqrt{\frac{4A}{\pi}},$$

where A is the computed area of the circle, is plotted as a function of time for different threshold values. The plot reveals that erosion will not happen for threshold values equal and greater than 5. In other words, each black pixel can only have maximum number of 4 white pixels within its neighbourhood.

As the value of threshold is decreased, the diameter decreases linearly as a function of time, the slope of the curve increases, and the time for complete erosion shortens. Notice that the curves for threshold values of 2 and 3 are identical.

Fig. 4 shows images of the circle as a function of time for threshold value equals to 4. The circle maintains its shape throughout the erosion steps until its diameter diminishes after 42 seconds.

Fig. 5 and Fig. 6 present the results for erosion of a square. The length, L, is defined by

$$L = \sqrt{A}$$

where A is the computed area of the square. Compared to Fig. 3, similar trend is shown in Fig. 5. However, for square, the curves for threshold values of 1, 2, and 3 are identical. Our simulation indicates that one layer of the outer pixel is eroded at each erosion step that results in preserving the square shape for these three threshold values.

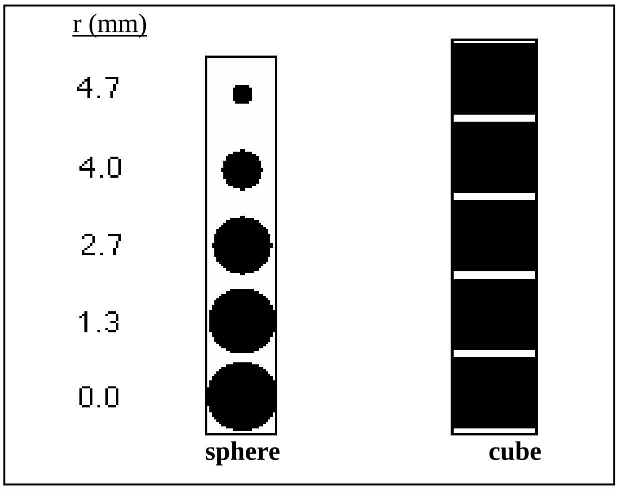

Fig. 2: Initial Shapes For 3-D Model

For threshold value of 4, images in Fig. 6 reveals that erosion happens only at the corner of the square during the first 6 seconds. After that the shape of the image becomes more like the circle for the rest of the process. The fact explains the shape of the corresponding curve in Fig. 5.

An important conclusion from this 2-D simulation concerns about appropriate value of the threshold to accurately describe real engineering problems. The value is best chosen after enough experimental data are collected and analysed. From previous data of dissolution [3, 5] and combustion processes [6, 7], the optimum threshold value is either 2 or 3. For these two values, it is very interesting to see that corresponding curves in Fig. 3 and Fig. 5 are identical as a function of time.

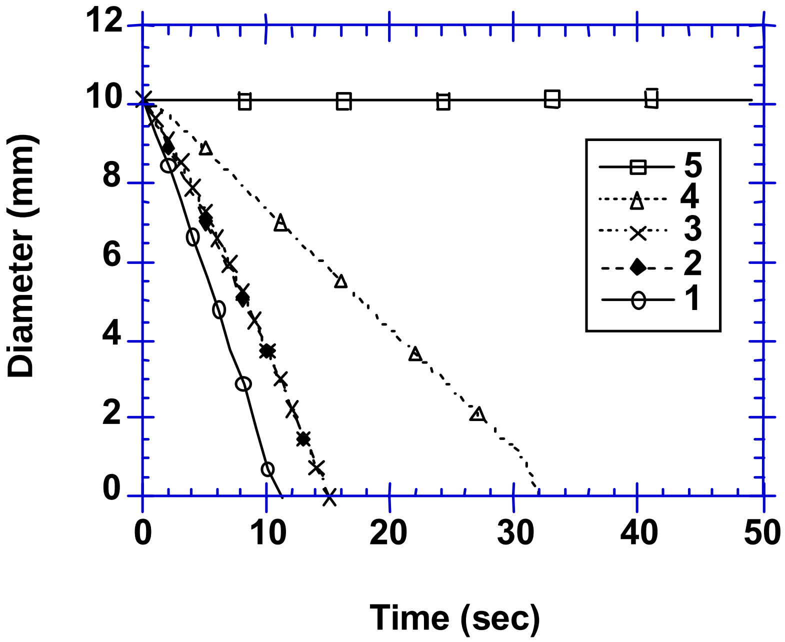

Fig. 3: Diameter of a circle as a function of time for various threshold values

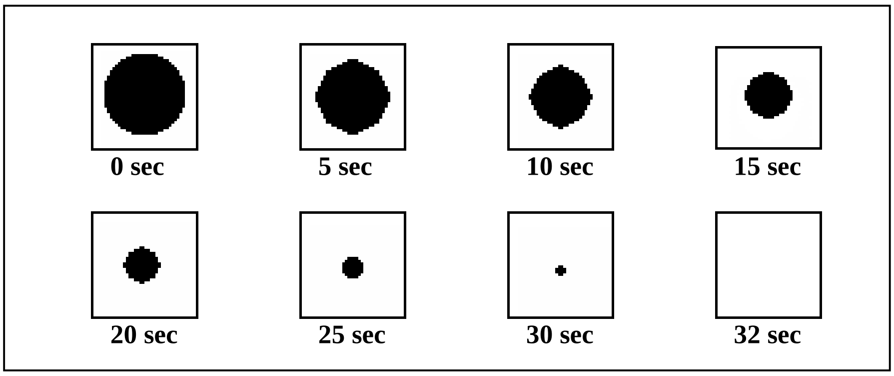

Fig. 4: Images of a circle as a function of time for threshold value equals to 4

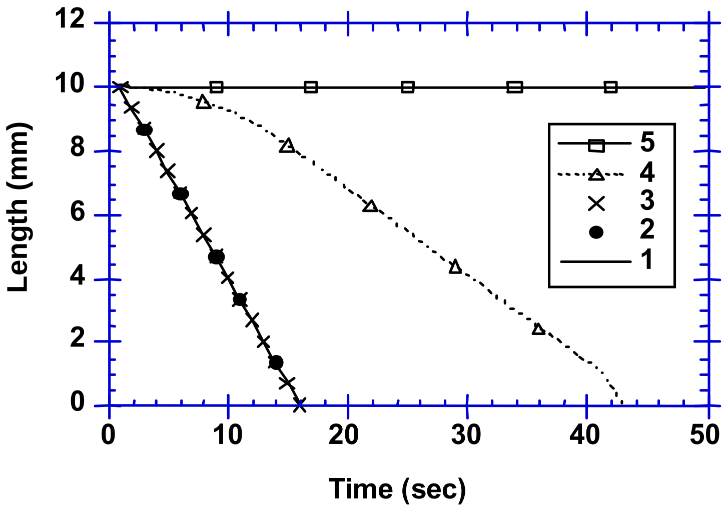

Fig. 5: Length of a square as a function of time for various threshold values

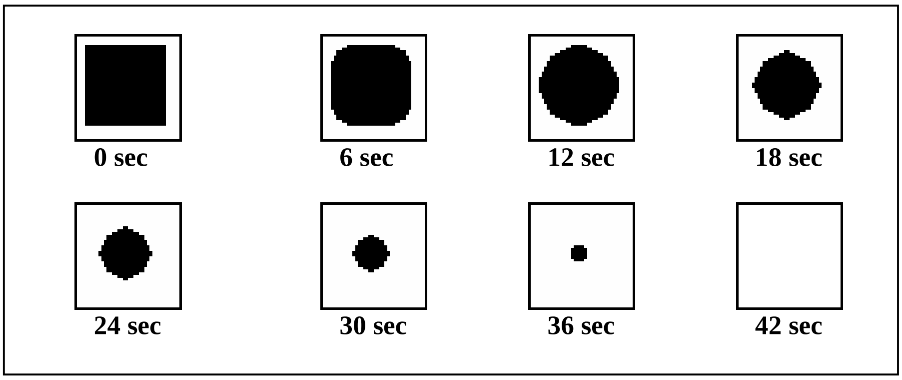

Fig. 6: Images of a square as a function of time for threshold value equals to 4

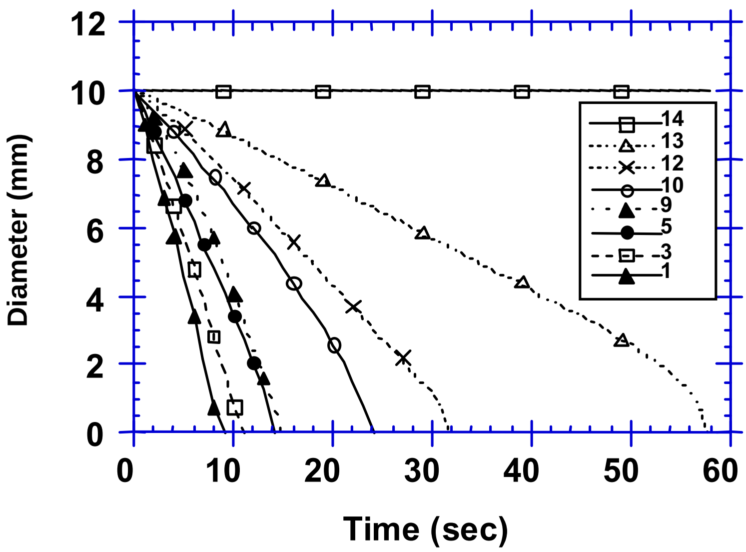

Fig. 7: Diameter of a sphere as a function of time for various threshold values

5.0 RESULTS AND DISCUSSION: 3-D MODEL

The threshold value may vary from 1 to 26 for 3-D model since a pixel has 26 neighbours in the surrounding. The results for a sphere presented in Fig. 7 indicates that erosion only occurs for threshold value less than 14. The equivalent diameter, D, is defined by

$$D = \sqrt[3]{\frac{6V}{\pi}},$$

where V is the computed volume of the sphere. Similar trend discussed for 2-D model is also observed here as the threshold value decreases. Fig. 8 shows the image of the sphere (at various distance from its centre, r) as a function of time. As time progresses, the images further from the centre begin to disappear as the sphere shrinks to zero radius after 58 seconds.

The results for 3-D simulation of a cube are shown in Fig. 9 and Fig. 10. The equivalent length, L, is defined by $L = \sqrt[3]{V}$. The plots in Fig. 9 indicates that the curves for threshold values of 1, 3, 5, and 9 are identical. The same is true for threshold values of 10, 11, and 12. The images reveal that the shape of the cube is preserved for the lower threshold values. For higher value, the four corners of the cube are eroded first and this is demonstrated in Fig. 10 for the threshold value of 13.

The results of 2-D and 3-D can be quantified by comparing the ratio of the threshold values to the total number of neighbour for each pixel. For the same ratio, plots for 2-D and 3-D are almost identical.

With this observation, the optimum number of threshold values in 3-D should be chosen to be between 6 to 9 in order to simulate real engineering problems involving dissolution or combustion processes.

6.0 CONCLUSIONS

Both 2-D and 3-D cellular models described in this paper are useful to simulate real physical engineering problems such as dissolution, crystal growth, combustion, and chemical vapour deposition.

The 3-D model is very much needed due to the fact that real problems do not always come in nice shape. In this paper, two shapes (sphere and cube) are tested using 3-D models and these shapes can also be simulated using 2-D (circle and square). However, for the shapes like cylinder, pyramid, and other irregular shape, the 3-D model is much more superior to get good simulation results.

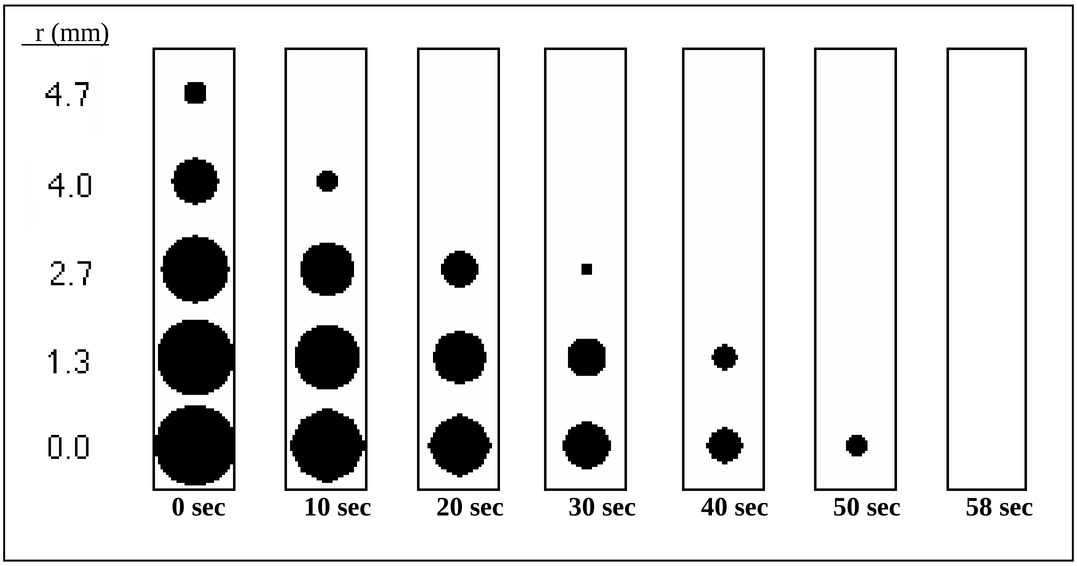

Fig. 8: Images of a sphere as a function of time for threshold value equals to 4 (r is the distance from the centre of the sphere)

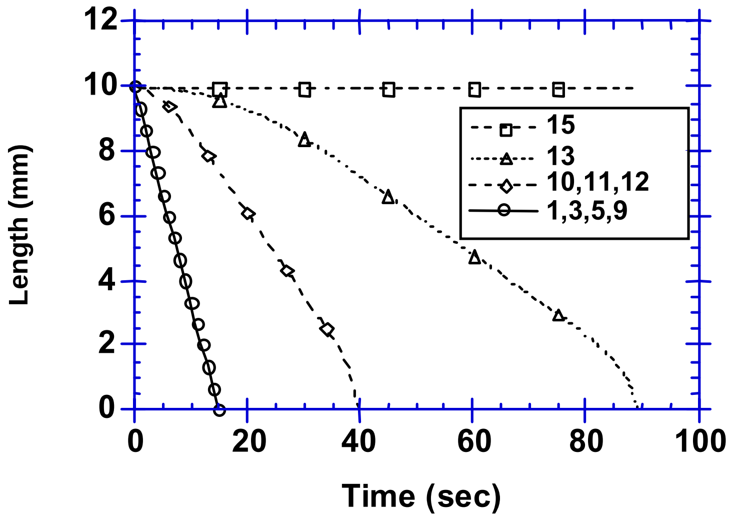

Fig. 9: Length of a cube as a function of time for various threshold values

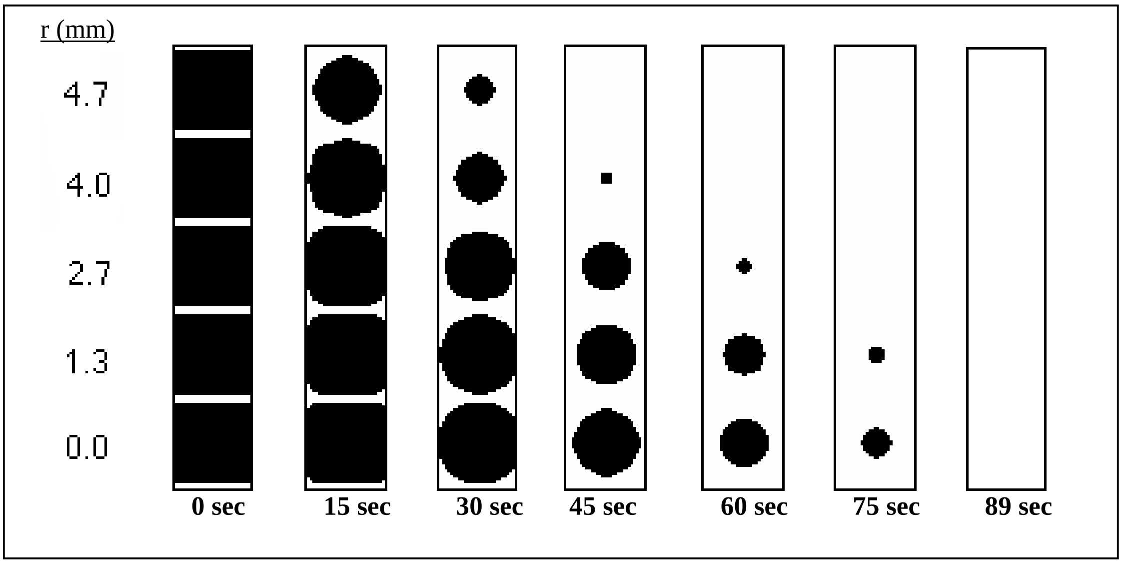

Fig. 10: Images of a cube as a function of time for threshold value equals to 4 (r is the distance from the middle of the cube)

7.0 ACKNOWLEDGMENT

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REFERENCES

[1] R. C. Gonzalez and R. E. Woods, *"Digital Image Processing"*, Addison-Wesley Publishing Company, 1993.

[2] W. K. Pratt, *"Digital Image Processing"*, John Wiley & Sons, Inc., 1991.

[3] Ahmad Faris Ismail, "Discrete Simulation Using Digital Image Processing and Mathematical Model of Dissolution Processes", *Malaysian Journal of Computer Science*, Vol. 7, 1994, pp. 51-57.

[4] J. Garside, J. W. Mullin and S. N. Das, "Importance of Crystal Shape in Crystal Growth Rate Determinations", *Ind. Eng. Chem. Process. Des. Develop. 12(3)*, 1973, pp. 369-371.

[5] A. A. Noyes and W. R. Whitney, "Rate of Solution of Solid substances in Their Own solution", *Z. Phys. Chem.,* Vol. 23, 1897, pp. 689-692.

[6] G. F. Froment and K. B. Bischoff, "*Chemical Reactor Analysis and Design*", John Wiley and Sons, Inc., 1979.

[7] S. K. Bhatia and D. D. Perlmutter, "A Random Pore Model for Fluid-Solid Reactions: I. Isothermal, Kinetic Control", *AIChE J., 26(3)*, 1980, pp. 379-386.

BIOGRAPHY

Dr. Ahmad Faris Ismail obtained his Ph.D from Rice University in 1993. Currently, he is an assistant professor at Faculty of Engineering, International Islamic University Malaysia. His research areas include discrete modeling with cellular automata, digital image analysis, and combustion. He is a member of AIChE and IEM.